Determining Measurement Uncertainty for Dimensional Measurements

The purpose of any measurement activity is to determine or quantify the size, location or amount of an object, substance or physical parameter of a feature we are measuring. We may be trying to determine the diameter of a wrist pin for an automobile engine (size), the X, Y and Z coordinates of a dowel pin hole relative to a reference surface in a die component (location) or the percentages of pigments in a specific shade of red paint (amount).

Following is an example of the equation of a dimensional measurement:
Equation #1: $Y = X + \sum C_i$

Where
Y = corrected value
X = measured value
C_i = corrections to the measured value

If we knew or could measure exactly all the elements that affect or alter the measured value we could add or subtract the appropriate corrections C_i and exactly adjust the measured value X to obtain the corrected value Y. However all of these elements have a certain degree of potential error or uncertainty associated with them and as a result the combined affect on the corrected value is that we can only express it as being a certain value within a certain range to a certain level of confidence. This resulting "range" is the *Expanded Uncertainty*.

The individual performing a measurement, the equipment used to take the measurement and the environment in which the measurement is taken are all potential sources of error (which produces uncertainty) in the measurement values obtained. Training of the individuals performing the measurement activity, careful selection of the equipment and specification of the procedures used to obtain measurements and control of the environment in which the measurements are taken enable us to reduce to a minimum the magnitude of measurement error.

The formal determination of a value for uncertainty of a measurement requires that we analyze and understand the interaction of the individual, the equipment and the environment to determine the manner in which they contribute to the measurement error and the expected magnitude of their contributions.

Expanded uncertainty is the extent to which the measured values we obtain can be expected to deviate from the actual value of the feature we are measuring. A stated value of expanded uncertainty is typically expressed at a 95% confidence level, which means that in 95 cases out of 100, the measurement error will not exceed the stated amount. In the example, which follows, we will calculate the Expanded Uncertainty of a 1-inch micrometer and although this example deals with dimensional measurement, the process of determination and the analytical methods involved are consistent for all measurement activities.

Begin the process of determination of uncertainty by establishing an uncertainty budget. This is done by identifying all significant elements of the measurement process and determining their individual contributions to the total uncertainty of the measurement. Reasoned judgments must be made as to which elements are considered significant and must therefore be included and which ones will result in contributions so small as to be insignificant and can safely be excluded.

The micrometer which is the subject of our determination has the following characteristics:

Range of measurement: 0.0 to 1.0 in / 0.0 to 25.4 mm
Resolution: 0.0001 in / 0.00254 mm
Type of measurement display: Digital
It is equipped with a "ratchet" or "clutch" to minimize inconsistency in applied measuring force.

The gage block which will serve as the "master" is made of a special grade of steel, which is capable of being hardened, and which will retain a high degree of dimensional stability.

For this specific micrometer, the following elements are identified as contributing to the overall measurement uncertainty and therefore requiring a correction to the measured value to account for their contribution.

1.) C_1 = Uncertainty of setting master
2.) C_2 = Uncertainty of repeatability
3.) C_2 = Uncertainty of resolution
4.) C_3= Uncertainty of thermometer
5.) C_4 = Uncertainty in CTE (Coefficient of Thermal Expansion)
6.) C_5 = Uncertainty due to temperature differential between micrometer and standard

Since the micrometer and the gage block are both made of steel which has a high compressive strength and the micrometer has a ratchet on the thimble to minimize applied measurement force, dimensional distortion of the micrometer frame and the gage block is considered insignificant and therefore no correction is deemed necessary. This statement is an example of reasoned judgment in determining what contributors are worthy of analysis and which ones can be excluded without impacting the final determination.

The uncertainty budget used has 9 columns for entering data (see attached spreadsheet). They are listed below with a brief explanation of what they represent.

Column 1.) The identifier for the correction being determined. C_1, C_2, C_3 etc.

Column 2.) The source of the uncertainty associated with the correction identifier.

Column 3.) The statement or estimate of the uncertainty associated with the source. This is typically an expanded uncertainty for Type B's and may be an expanded or standard uncertainty for Type A's. It is necessary to determine which uncertainty Is given for Type A's. The units are typically micro inches (µin) or micrometers

Column 4.) Degrees of Freedom. This is an indicator of how much information was used in the statistical determination of an uncertainty estimate. It is equal to the number of measurements taken (n) – 1. If 30 measurements were taken then the degrees of freedom would be (n – 1) or 30 – 1 = 29. For Type B uncertainties the degrees of freedom are by convention considered to be (∞) infinity.

Column 5.) Type of uncertainty. Type A uncertainties are those that are statistically determined. Type B uncertainties are those that are determined by any other means. These means may include previous measurement data, experience with or general knowledge of the relevant materials or instruments, data provided in calibration and other reports, manufacturer's specifications and uncertainties assigned to reference data taken from handbooks.

Column 6.) Type of distribution. Each contributor to the overall uncertainty has an underlying distribution associated with the various values that might be obtained for it. There are numerous types of distributions these values might take but for dimensional uncertainties we will consider the 4 most common types as listed below.

Normal distribution:
This distribution results from processes that produce a population in which 68.26% of the population is located within +/-1 Σ of the mean, 95.45% is located within +/-2 Σ of the mean and 99.73% is located within +/-3 Σ from the mean.

Rectangular distribution:
This distribution results from processes that produce a population in which all values are equally likely to occur. 95% of the population is contained within +/- $\sqrt{3}\,\Sigma$.

Triangular distribution:
This distribution results from processes that produce a population in which most values are concentrated near the center. 95% of the population is contained within +/- $\sqrt{6}\,\Sigma$.

"U" distribution:
This distribution results from processes that produce a population in which most values are concentrated near the boundaries. 95% of the population is contained within +/- $\sqrt{2}\,\Sigma$.

Column 7.) Divisor. This is the number that divides the *Expanded Uncertainty* to obtain the *Standard Uncertainty*. It is the coefficient of S in the above descriptions.

Column 8.) Standard Uncertainty. If the stated or estimated uncertainty in column 3 is an Expanded uncertainty then divide it by the divisor in column 7 to obtain the Standard uncertainty and enter it in this column. If the uncertainty in column 3 is a standard uncertainty then transfer it directly to this column. The units should be the same as column 3.

Column 9.) Variance. The value in this column is the square of the Standard uncertainty found in column 8. (Variance = Standard Uncertainty^2). The units should be μin^2.

Below column 3 is a cell titled Sum of Variances. Place in this cell the sum of all the variances in column 9.

Below the cell referenced above is a cell titled Combined Standard Uncertainty (U_c). The Combined Standard Uncertainty is the square root of the Sum of the Variances. Obtain this value and place it in this cell.

Below this cell is a cell titled Expanded Uncertainty (U). $U = kU_c$ where k is a coverage factor determined as follows.

Rules for selecting values of the coverage factor *k*: (This list is not intended to be all inclusive)

1.) If one or two contributors are dominant and their distribution is rectangular use k=1.65 for a result at a 95% confidence level.

2.) If three or more contributors are dominant and their distribution is rectangular use k = 2.00 for a result at a 95% confidence level.

3.) If one contributor is dominant and its distribution is normal use k = 2.00 for a result at a 95% confidence level. If the contributor is Type A and the degrees of freedom are small (less than 30) obtain the value of k from a table of values for Student's *t*-Distribution using the appropriate degrees of freedom at a 95% confidence level.

4.) If a combination of contributors with normal and rectangular distributions is dominant use k = 2.00 for a result at a 95% confidence level.

Data Collection

Uncertainty of Master: The master is a 1.00 in grade 2 gage block. The calibration certificate for the gage block indicates that it has an expanded uncertainty of <u>3.00 µin</u>. Since the standard uncertainty is not determined statistically, there are no degrees of freedom. For the same reason it is a <u>Type B</u> uncertainty. Gage blocks are sorted for size and categorized into grades based on accuracy. The most accurate are grade 1, the next level of accuracy is grade 2 etc. This process tends to produce (in all classes except the most accurate and the least accurate) a "<u>U</u>" distribution where the majority of items will be near the boundaries and few toward the center. The divisor for a "U" distribution is $\underline{\sqrt{2}}$. The standard uncertainty is <u>2.12</u>. The variance is <u>4.49</u>

Determine the error due to repeatability and the error due to resolution. Use whichever is greater in the final determination of Expanded Uncertainty.

Uncertainty of Repeatability: 30 measurements of the gage block were taken with the micrometer and the standard deviation (standard error) of these values was determined to be <u>38 µin</u>. Since this value is determined statistically, it has degrees of freedom equal to *n*-1 or <u>29</u>. Also since it is statistically determined it is a <u>Type A</u> uncertainty. The measurements produce a "<u>Normal</u>" distribution and since the standard error was determined the divisor is <u>1.00</u>. The variance is <u>1,444.00</u>

Uncertainty of Resolution: The micrometer reads to the nearest <u>50 µin</u>. The expanded uncertainty is one half of the resolution, <u>25 µin</u>. This is a <u>Type B</u> uncertainty with no degrees of freedom since it is not determined statistically. The distribution is "<u>Rectangular</u>" since the actual reading that is rounded off to determine the displayed value is equal to the displayed value +/-2.5 µin and all 5 digits have an equal likelihood of occurrence. The divisor for a "Rectangular" distribution is $\underline{\sqrt{3}}$. The standard uncertainty is <u>14.43</u>. The variance is <u>208.33</u>.

Next we need to examine thermal conditions in the lab and thermal characteristics of the materials involved in the uncertainty determination. The laboratory is maintained at 68°F +/-1°F. The heating and cooling system in the laboratory is controlled by a thermostat that turns on the heat when the temperature approaches 67°F and turns on the Air-conditioning when the temperature approaches 69°F

Uncertainty of Thermometer: As stated, the laboratory temperature is controlled to +/-1°F however the thermometer is not exact and we need to determine the uncertainty associated with it. Expect that the thermometer is accurate to within +/-.5°F. The laboratory temperature range is 68°F +/-1.5°F. The micrometer and the gage block are made of steel and the commonly stated value for CTE is 6 µin per inch of length per degree Fahrenheit. We will use the following equation to calculate the resulting uncertainty associated with the inaccuracy of the thermometer.

Equation #2: $\Delta L = L * \Delta T * \alpha$
Where…
ΔL = the error in length
L = the nominal length of the gage block
ΔT = the expected error of the thermometer in degrees Fahrenheit
α = CTE (Coefficient of Thermal Expansion)
Substituting actual values:
ΔL = 1.00*.5*6.00 µin =3.00 µin

The expanded uncertainty is 3.00 µin. It is a Type B uncertainty therefore there are no degrees of freedom. It is unlikely that the worst case as calculated will always happen so consider this as a "Rectangular" distribution for which the divisor is $\sqrt{3}$. The standard uncertainty is 1.73. The variance is 3.00.

Uncertainty of CTE (Coefficient of Thermal Expansion): The micrometer and the gage block are made of steel and the commonly stated value for CTE is 6 µin per inch of length per degree Fahrenheit. Although this value is satisfactory for most engineering calculations, it is not an exact value and when considering precise measurements it is necessary to consider the uncertainty associated with the value for CTE. It is reasonable to expect that this value might vary by as much as 15% or .9 µin per inch of length per degree Fahrenheit. To calculate the resulting uncertainty associated with the CTE we will use Equation #3 below.

Equation #3: $\Delta L = L * \Delta T * \Delta\alpha$
Where…
ΔL = the error in length
L = the nominal length of the gage block
ΔT = the deviation from standard temperature in the lab
$\Delta\alpha$ = the difference in CTE of the gage block and the micrometer
Substituting actual values:
ΔL = 1.00*1.5*.9 µin = 1.35 µin

The expanded uncertainty is 1.35 µin. It is a Type B uncertainty therefore there are no degrees of freedom. Again it is unlikely that the worst case as calculated will always happen so consider this as a "Rectangular" distribution for which the divisor is $\sqrt{3}$. The standard uncertainty is 0.78. The variance is 0.61.

Uncertainty due to temperature differential: If the temperature of the micrometer and the gage block are not the same they will experience unequal thermal expansion or contraction and this will produce an error in the measurement. Laboratory procedures must address the need for both items to be at the same temperature. Perhaps a policy states that items for test must be placed in the controlled environment of the laboratory a minimum of 24 hours prior to measurement so that thermal equilibrium may be attained. Small fluctuations are still possible as the temperature control system of the laboratory makes slight adjustments to maintain the stated temperature of 68°F +/-1°F. It is reasonable to expect that under laboratory conditions the temperature differential does not exceed 0.1°F. We will use Equation #2 below to calculate this error as follows.

Equation #2: $\Delta L = L * \Delta T * \alpha$
Where…
ΔL = the error in length

L = the nominal length of the gage block
ΔT = the temperature differential between the micrometer and the gage block
α = CTE (Coefficient of Thermal Expansion)

Substituting actual values:

ΔL = 1.00*0.1*6 µin = 0.60 µin

The expanded uncertainty is 0.60 µin. It is a Type B uncertainty therefore there are no degrees of freedom. Again it is unlikely that the worst case as calculated will always happen so consider this as a "Rectangular" distribution for which the divisor is $\sqrt{3}$. The standard uncertainty is 0.35. The variance is 0.12.

Summing all of the variances:

Variance of Master:	4.49
(Larger of repeatability vs. resolution)	
Variance of Repeatability:	1,440.00
Variance of uncertainty of thermometer:	3.00
Variance of uncertainty in CTE:	.61
Variance uncertainty due to temperature differential:	.12
$\sum$ Variances =	1,448.22

Combined standard uncertainty: $\sqrt{1448.22} = 38.05$

Referring to the rules for selecting a coverage factor, use k=2.00 per rule number 3 since the one contributor is obviously dominant and it is normally distributed.

Expanded uncertainty (Note: k=2.00): 76.11 µin

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Henry Alexander (PJLA Technical Committee Member)
Henry Alexander Engineering, Inc.
1367 Snyder Road
Norwalk, OH 44857

Approved by PJLA